Wavelet-Reduced Order Modelling

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Over the years computational homogenisation (CH), schematically shown in Figure 1, has proven to be an accurate framework to derive the macroscopic material properties emerging from micro-structures. Unfortunately the homogenisation procedure is computationally expensive for nonlinear micro-structural models due to the repetitive evaluation of the micro-scale problem. Due to the nested Newton-Raphson schemes of the micro- and macroscopic problems, the size of the system of equations scales quadratically with the number of dofs involved. When considering material models with history parameters, it is required to track the history for all integration points in all microstructures. These computational framework quickly becomes intractable for large micro- or macro-scale problems in terms of available computation time and memory.

To restrict the computational time and memory usage, the microstructural model is reduced using a more optimal global basis constructed from the physical modes occurring in the microstructure, schematically depicted in Figure 2. The number of physical modes present in a micro-structural problem is often much lower than the number of degrees of freedom used in a traditional discretisation. The physical modes of the solution are derived a-priori by obtaining snapshots of solutions and performing a Proper Orthogonal Decomposition to distill only essential modes.

Applying a Bubnov-Galerkin discretisation using the reduced basis on the weak form of the linear momentum balance yields an integral over the complete microstructure. This integral is unreduced and therefore costly. In this work a wavelet-based integration scheme will be presented to reduce the cost of integration of the weak-form while maintaining control over the error made by adaptively refining where the solution requires it. A comparison between the traditional full-order solution, the standard reduced order solution and the wavelet-reduced order model (W-ROM) will be presented with emphasis on the error in both the solution on the microscale level and the derived macroscopic properties of the material.

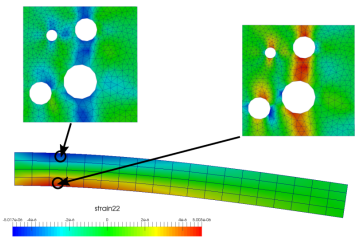

Figure 1: Computational homogenisation of the micro-structure to resolve a cantilever beam

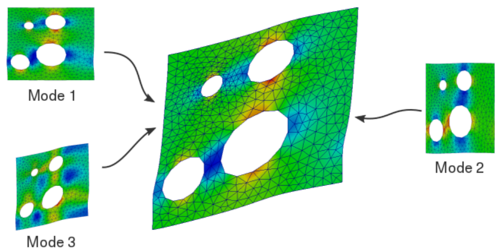

Figure 2: Reduced Order Modelling of the microstructure